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Review Article

Improved Ratio cum Product Estimators for Finite Population Mean with Known Quartiles and Their Functions

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Abstract

In sample surveys auxiliary information is quite often available from previous experience and using the auxiliary information several estimators are introduced like Ratio estimator modified ratio and product estimators for the estimation of population mean. This manuscript deals with the improvements on ratio cum product estimators for the estimation of population mean of the study variable by using the known values of quartiles of the auxiliary variable and their functions. The bias and mean squared error of proposed estimators are obtained. An empirical condition is developed to assess the performance of proposed estimators over the existing estimators. A numerical study is carried out to assess the efficiency of proposed estimators over the existing estimators with the help of some known natural populations. Based on the simulation study and numerical studies, the proposed estimators are less biased (almost unbiased) and it performs better than the existing estimators.

Introduction

In sample survey auxiliary information is quite often available from previous experience. This information is used in the estimation stage. The efficiency of the estimators of the population parameters can be increased by using the prior information of the study characteristics. In literature several estimators exist with auxiliary variables. Using auxiliary information the different types of estimators and their modifications are widely used for the estimation of population mean. When the auxiliary information is to be used in the estimation stage ratio, product and regression methods are widely used.The commonly used the population parameters of the auxiliary variables are mean, median, coefficient of variation, coefficient of skewness, coefficient of kurtosis etc. Ratio method of estimation is extensively used because of its computational simplicity and applicability. If the correlation between study variable and auxiliary variables are positive the ratio method of estimation is used and the correlation between study and auxiliary variables are negative the product method of estimation is used. Several researchers have directed their efforts towards to get efficient estimators of population mean. These estimators are biased but the percentage relative efficiency is better than that of simple random sampling, ratio and product estimators. We know that quartile values and their functions are unaffected by extreme values. For this reason we consider the problem of estimation of population mean of study variable using quartile values and their functions of the auxiliary variable. So we have suggested new modified ratio cum product estimators for estimating the population mean of the study variable. To know more about historical developments of the estimation of population mean, the readen are referred to Cochran [1,2], Khan M and Sabbir J [3] , J. Subramani and Master Ajith [4,5], Khoshnevisan.et.al [6], Murthy [7,8], R.Tailor and B. Sharma [9], Singh et.al [10], Sisodia and Dwivedi [11], Subramani [12], Subramani and Kumarapandiyan Subramani [13,14], Upadhyaya and Singh [15], Yan and Tian [16] and the references cited there in.etc.

Notations and Literature Review

Let $U=\{U_1,U_2,.......,U_N\}$ be a finite population having N units. Each $U_i=(X_i,Y_i), i=1,2,.....N$ has a pair of values. Here Y be the study variable and X be the auxiliary variable which is correlated with Y. Let $y=\{y_1,y_2,....,y_n\}$ and $x=\{x_1,x_2....,x_n\}$ be n sample values. Then $\bar{y}$ and $\bar{x}$ be the sample means of the study and auxiliary variables, $S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2, S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2$ and $S_{xy}=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})(X_i-\bar{X})$ be the population variance and co-variance of the study variable and auxiliary variable. Similarly the coefficient of variations and coefficient of co-variance of these variables are defined as $C_x=\frac{S_y}{\bar{X}}, C_y=\frac{S_x}{\bar{Y}}$, and $C_{xy}=\frac{S_{xy}}{\bar{X}\bar{Y}}=\rho C_x C_y$ where ρ is the correlation coefficient. The quartile values and their functions are unaffected by the presence of outliers in the population values. There are three quartiles, first quartile Q_1, second quartile Q_2 or median and the third quartile Q_3. In the first quartile or lower quartile 25% of observations are less than it and in the third quartile or upper quartile 75 % of observations are less than it. The inter quartile range is another measure of quartiles defined as $Q_r=Q_3-Q_1$. Semi inters quartile range or quartile deviation is another measure of dispersion. It is

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the one half of inter quartile range, defined as $Q_d = \frac{Q_3 - Q_1}{2}$ And the quartile averages is defined as $Q_a = \frac{Q_3 + Q_1}{2}$.

In simple random sampling without replacement, the estimator of population mean $\bar{y}_{srs}$ is an unbiased estimator for the population mean $\bar{Y}$ and its variance is

$$v(\bar{y}_{srs}) = \delta \bar{Y}^2 C_y^2 \quad (1)$$

Where $\delta = \frac{1-f}{n}$ is the finite population correction?

The ratio estimator Cochran [1] is given

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \bar{X} = \hat{R}\bar{X}$$

The bias and mean squared error of ratio estimator up to first order approximations are

$$B(\hat{\bar{Y}}_R) = \delta \bar{Y}(C_x^2 - \rho C_x C_y)$$

$$MSE(\hat{\bar{Y}}_R) = \delta \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_x C_y) \quad (2)$$

The modified ratio estimators for the population mean $\bar{Y}$ with known quartiles and their functions proposed by Al- Omari et al. [17] and Subramani J and G. Kumarapandiyan [13] are given in the table 1

The auxiliary variable and study variable are negatively correlated; the product estimator and its modifications are used. The product estimator Murthy [7] is given by

$$\hat{\bar{Y}}_P = \bar{y} \frac{\bar{x}}{\bar{X}}$$

The bias and mean squared error of the product estimator are given by

$$B(\hat{\bar{Y}}_R) = \delta \bar{Y}(\rho C_x C_y)$$

$$MSE(\hat{\bar{Y}}_R) = \delta \bar{Y}^2 (C_y^2 + C_x^2 + 2\rho C_x C_y) \quad (3)$$

The modified product estimators for the population mean $\bar{Y}$ with known quartiles and their functions are given in the table 2.

Suggested Class of Estimators

We have suggested class of ratio cum product estimators for the population mean by using the known population quartiles and their functions of the auxiliary variable X.

The proposed class of ratio cum product estimators with known quartiles and their functions are given as

$$\hat{\bar{Y}}_{P1} = \alpha_1 \lambda_1 \bar{y}(\frac{\bar{X}+Q_1}{\bar{x}+Q_1}) + (1-\alpha_1)\gamma_1 \bar{y}(\frac{\bar{x}+Q_1}{\bar{X}+Q_1})$$

$$\hat{\bar{Y}}_{P2} = \alpha_2 \lambda_2 \bar{y}(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}) + (1-\alpha_2)\gamma_2 \bar{y}(\frac{\bar{x}+Q_3}{\bar{X}+Q_3})$$

$$\hat{\bar{Y}}_{P3} = \alpha_3 \lambda_3 \bar{y}(\frac{\bar{X}+Q_r}{\bar{x}+Q_r}) + (1-\alpha_3)\gamma_3 \bar{y}(\frac{\bar{x}+Q_r}{\bar{X}+Q_r})$$

$$\hat{\bar{Y}}_{P4} = \alpha_4 \lambda_4 \bar{y}(\frac{\bar{X}+Q_d}{\bar{x}+Q_d}) + (1-\alpha_4)\gamma_4 \bar{y}(\frac{\bar{x}+Q_d}{\bar{X}+Q_d})$$

$$\hat{\bar{Y}}_{P5} = \alpha_5 \lambda_5 \bar{y}(\frac{\bar{X}+Q_a}{\bar{x}+Q_a}) + (1-\alpha_5)\gamma_5 \bar{y}(\frac{\bar{x}+Q_a}{\bar{X}+Q_a})$$

Where $\lambda_i = \frac{S_y}{S_y + a_i C_y}$, $\gamma_i = \frac{S_y}{S_y + b_i C_y}$ i = 1, 2,3,4,5. Here a_i's and b_i's are constants

Table 1: Bias and MSE of existing modified ratio estimators.

Existing Estimators	Constants	Bias	Variance/Mean squared Error
$\hat{\bar{Y}}_{MR1} = \bar{y}(\frac{\bar{X}+Q_1}{\bar{x}+Q_1})$ Al- Omari et al	$\theta_1 = \frac{\bar{X}}{\bar{X}+Q_1}$	$B(\hat{\bar{Y}}_{MR1}) = \delta\bar{Y}(\theta_1^2 C_x^2 - \rho\theta_1 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR1}) = \delta\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\rho\theta_1 C_x C_y)$
$\hat{\bar{Y}}_{MR2} = \bar{y}(\frac{\bar{X}+Q_3}{\bar{x}+Q_3})$ Al- Omari et al	$\theta_2 = \frac{\bar{X}}{\bar{X}+Q_3}$	$B(\hat{\bar{Y}}_{MR2}) = \delta\bar{Y}(\theta_2^2 C_x^2 - \rho\theta_2 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR2}) = \delta\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\rho\theta_2 C_x C_y)$
$\hat{\bar{Y}}_{MR3} = \bar{y}(\frac{\bar{X}+Q_r}{\bar{x}+Q_r})$ Subramani J. and G. Kumarapandiyan	$\theta_3 = \frac{\bar{X}}{\bar{X}+Q_r}$	$B(\hat{\bar{Y}}_{MR3}) = \delta\bar{Y}(\theta_3^2 C_x^2 - \rho\theta_3 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR3}) = \delta\bar{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\rho\theta_3 C_x C_y)$
$\hat{\bar{Y}}_{MR3} = \bar{y}(\frac{\bar{X}+Q_d}{\bar{x}+Q_d})$ Subramani J. and G. Kumarapandiyan	$\theta_4 = \frac{\bar{X}}{\bar{X}+Q_d}$	$B(\hat{\bar{Y}}_{MR4}) = \delta\bar{Y}(\theta_4^2 C_x^2 - \rho\theta_4 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR4}) = \delta\bar{Y}^2(C_y^2 + \theta_4^2 C_x^2 - 2\rho\theta_4 C_x C_y)$
$\hat{\bar{Y}}_{MR5} = \bar{y}(\frac{\bar{X}+Q_a}{\bar{x}+Q_a})$ Subramani J. and G. Kumarapandiyan	$\theta_5 = \frac{\bar{X}}{\bar{X}+Q_a}$	$B(\hat{\bar{Y}}_{MR5}) = \delta\bar{Y}(\theta_5^2 C_x^2 - \rho\theta_5 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR5}) = \delta\bar{Y}^2(C_y^2 + \theta_5^2 C_x^2 - 2\rho\theta_5 C_x C_y)$

Table 2: Bias and MSE of Existing modified product estimators.

Existing Estimators	Constants	Bias	Variance/Mean squared Error
$\hat{\bar{Y}}_{MR1} = \bar{y}(\frac{\bar{X}+Q_1}{\bar{x}+Q_1})$	$\theta_1 = \frac{\bar{X}}{\bar{X}+Q_1}$	$B(\hat{\bar{Y}}_{MP1}) = \delta\bar{Y}(\rho\theta_1 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR1}) = \delta\bar{Y}^2(C_y^2+\theta_1^2 C_x^2 - 2\rho\theta_1 C_x C_y)$
$\hat{\bar{Y}}_{MR2} = \bar{y}(\frac{\bar{X}+Q_3}{\bar{x}+Q_3})$	$\theta_2 = \frac{\bar{X}}{\bar{X}+Q_3}$	$B(\hat{\bar{Y}}_{MR2}) = \delta\bar{Y}(\theta_2^2 C_x^2 - \rho\theta_2 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR2}) = \delta\bar{Y}^2(C_y^2+\theta_2^2 C_x^2 - 2\rho\theta_2 C_x C_y)$
$\hat{\bar{Y}}_{MR3} = \bar{y}(\frac{\bar{X}+Q_r}{\bar{x}+Q_r})$	$\theta_3 = \frac{\bar{X}}{\bar{X}+Q_r}$	$B(\hat{\bar{Y}}_{MR3}) = \delta\bar{Y}(\theta_3^2 C_x^2 - \rho\theta_3 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR3}) = \delta\bar{Y}^2(C_y^2+\theta_3^2 C_x^2 - 2\rho\theta_3 C_x C_y)$
$\hat{\bar{Y}}_{MR3} = \bar{y}(\frac{\bar{X}+Q_d}{\bar{x}+Q_d})$	$\theta_4 = \frac{\bar{X}}{\bar{X}+Q_d}$	$B(\hat{\bar{Y}}_{MR4}) = \delta\bar{Y}(\theta_4^2 C_x^2 - \rho\theta_4 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR4}) = \delta\bar{Y}^2(C_y^2+\theta_4^2 C_x^2 - 2\rho\theta_4 C_x C_y)$
$\hat{\bar{Y}}_{MR5} = \bar{y}(\frac{\bar{X}+Q_a}{\bar{x}+Q_a})$	$\theta_5 = \frac{\bar{X}}{\bar{X}+Q_a}$	$B(\hat{\bar{Y}}_{MR5}) = \delta\bar{Y}(\theta_5^2 C_x^2 - \rho\theta_5 C_x C_y)$	$MSE(\hat{\bar{Y}}_{MR5}) = \delta\bar{Y}^2(C_y^2+\theta_5^2 C_x^2 - 2\rho\theta_5 C_x C_y)$

The Bias and Mean Squared error of the Proposed Estimators

To obtain the bias and mean squared error of the proposed estimators,

Consider, $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$ $\theta_i = \frac{\bar{X}}{\bar{X}+T_i}, T_1 = Q_1, T_2 = Q_3, T_3 = Q_r, T_4 = Q_d, T_5 = Q_a$

$$E(e_0) = E(e_1) = 0, E(e_0^2) = \delta\bar{Y}^2 C_y^2, E(e_1^2) = \delta\bar{X}^2 C_x^2, E(e_0 e_1) = \delta\rho C_x C_y$$

Substitute the values of e_0 and e_1 in the proposed class of estimators and neglecting the high order expressions, we get

$$B(\hat{\bar{Y}}_{Pi}) = E(\hat{\bar{Y}}_{Pi} - \bar{Y})$$

$$B(\hat{\bar{Y}}_{Pi}) = \bar{Y}(\alpha_i\lambda_i + (1-\alpha_i)\gamma_i - 1) + \alpha_i\lambda_i B(\hat{\bar{Y}}_{MRi}) + (1-\alpha_i)\gamma_i B(\hat{\bar{Y}}_{MPi})$$

Where $B(\hat{\bar{Y}}_{MRi}) = \delta\bar{Y}(\theta_i^2 C_x^2 - \theta_i\rho C_x C_y)$, and

$$B(\hat{\bar{Y}}_{MPi}) = \delta\bar{Y}(\theta_i\rho C_x C_y) \text{ , i=1, 2,3,4,5,}$$

$$\delta = \frac{1-f}{n}, \theta_1 = \frac{\bar{X}}{\bar{X}+Q_1}, \theta_2 = \frac{\bar{X}}{\bar{X}+Q_3}, \theta_3 = \frac{\bar{X}}{\bar{X}+Q_r}, \theta_1 = \frac{\bar{X}}{\bar{X}+Q_d}, \theta_1 = \frac{\bar{X}}{\bar{X}+Q_a}$$

Where λ_i and γ_i are as defined above. If we assume that $a_i = 0, b_i = 0$ and $\alpha_i = 1$ then the proposed estimators are exactly equal to the estimators given in Table 3. If $a_i = 0, b_i = 0$ and $\alpha_i = 0$ then the proposed estimators are exactly equal to the estimators given in Table 4. If we assume that $a_i = B(\hat{\bar{Y}}_{MRi}), b_i = B(\hat{\bar{Y}}_{MPi})$ and $\alpha_i = 1$ or $\alpha_i = 0$ then the proposed estimators are almost unbiased ratio or product estimators corresponding to the estimators given in Tables 4 and 5. The final expression is obtained with only first-order approximation in the Taylor series expansion as,

$$MSE(\hat{\bar{Y}}_{Pi}) = E(\hat{\bar{Y}}_{Pi} - \bar{Y})^2$$

$$MSE(\hat{\bar{Y}}_{Pi}) = \bar{Y}^2(\alpha_i\lambda_i + (1-\alpha_i)\gamma_i - 1)^2 + \alpha_i^2\lambda_i^2(MSE(\hat{\bar{Y}}_{MRi}) + 2\bar{Y}B(\hat{\bar{Y}}_{MRi}))$$
$$+(1-\alpha_i)^2\gamma_i^2(MSE(\hat{\bar{Y}}_{MPi}) + 2\bar{Y}B(\hat{\bar{Y}}_{MPi})) - 2\bar{Y}(\alpha_i\lambda_i B(\hat{\bar{Y}}_{MRi}) + (1-\alpha_i)\gamma_i B(\hat{\bar{Y}}_{MPi}))$$
$$+2\alpha_i\lambda_i(1-\alpha_i)\gamma_i V(\bar{y}_{srs})$$

Where $B(\hat{\bar{Y}}_{MRi}) = \delta\bar{Y}(\theta_i^2 C_x^2 - \theta_i\rho C_x C_y), B(\hat{\bar{Y}}_{MPi}) = \delta\bar{Y}(\theta_i\rho C_x C_y)$

$$MSE(\hat{\bar{Y}}_{MRi}) = \delta\bar{Y}^2(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i\rho C_x C_y), MSE(\hat{\bar{Y}}_{MPi}) = \delta\bar{Y}^2(C_y^2 + \theta_i^2 C_x^2 + 2\theta_i\rho C_x C_y)$$

and $V(\bar{y}_{srs}) = \delta\bar{Y}^2 C_y^2$

If $a_i = B(\hat{\bar{Y}}_{MRi}), b_i = B(\hat{\bar{Y}}_{MPi})$ and α_i is optimum, then the proposed estimators are less bias (almost unbiased) ratio cum product estimators. The optimal value of α_i is determined by minimizing the $MSE(\hat{\bar{Y}}_{Pi})$ with respect to α_i for this differentiate MSE with respect to α_i and equate to zero (Table 6).

$\frac{\partial MSE}{\partial \alpha_i} = 0$, and we get the value of α_i, as

$$\alpha_i = \frac{\bar{Y}^2(\gamma_i - 1)(\gamma_i - \lambda_i) + \gamma_i^2\{(MSE(\hat{\bar{Y}}_{MPi}) + 2\bar{Y}B(\hat{\bar{Y}}_{MPi})) + \bar{Y}(\lambda_i B(\hat{\bar{Y}}_{MRi}) - \gamma_i B(\hat{\bar{Y}}_{MPi}))\} - \lambda_i\gamma_i V(\bar{y}_{srs})}{\bar{Y}^2(\lambda_i - \gamma_i)^2 + \lambda_i^2(MSE(\hat{\bar{Y}}_{MRi}) + 2\bar{Y}B(\hat{\bar{Y}}_{MRi})) + \gamma_i^2(MSE(\hat{\bar{Y}}_{MPi}) + 2\bar{Y}B(\hat{\bar{Y}}_{MPi})) - 2\lambda_i\gamma_i V(\bar{y}_{srs})}$$

Table 3: Constants and parameters of the population.

Constants	Population 1					Population 2				
	1	2	3	4	5	1	2	3	4	5
a_i	-0.048	-0.052	0.079	-0.050	0.051	-0.354	-0.332	-0.537	-0.564	-0.343
b_i	0.167	0.155	0.296	0.161	0.277	0.401	0.373	0.666	0.711	0.386
λ_i	1.000	1.001	0.999	1.000	0.999	0.976	0.997	0.987	0.971	0.990
γ_i	0.998	0.998	0.997	0.998	0.997	0.968	0.978	0.973	0.966	0.974
α_i	1.200	1.253	0.894	1.227	0.921	0.783	0.919	0.833	0.767	0.850
θ_i	0.526	0.489	0.933	0.507	0.874	0.680	0.449	0.570	0.726	0.541

Table 4: Bias and MSE of Proposed Class of Estimators.

Proposed Estimator	Population 1		Population 2	
	Bias	MSE	Bias	**MSE**
$\hat{\bar{Y}}_{P1}$	-1.01e-14	32.0836	-2.44e-15	2902.789
$\hat{\bar{Y}}_{P2}$	-1.31e-14	32.0898	-1.65e-14	3056.961
$\hat{\bar{Y}}_{P3}$	4.39e-15	31.9599	-6.88e-15	2985.792
$\hat{\bar{Y}}_{P4}$	-1.20e-14	32.0869	2.02e-14	2862.811
$\hat{\bar{Y}}_{P5}$	3.19e-15	31.9858	5.44e-15	3004.384

Table 5: Bias and MSE of Existing Estimators.

Estimator	Population 1		Population 2	
	Bias	MSE	Bias	**MSE**
$\bar{y}_{srs}$	-	55.6603	-	3849.248
$\hat{\bar{Y}}_{R}$	0.1131	35.0447	15.1642	4925.325
$\hat{\bar{Y}}_{P}$	0.3171	163.283	9.7687	12718.48
$\hat{\bar{Y}}_{MR1}$	-0.0477	33.9712	4.8929	3499.732
$\hat{\bar{Y}}_{MR2}$	-0.0522	34.7135	0.6462	3102.441
$\hat{\bar{Y}}_{MR3}$	0.0785	33.6996	2.5294	3243.522
$\hat{\bar{Y}}_{MR4}$	-0.0502	34.3409	6.0489	3641.236
$\hat{\bar{Y}}_{MR5}$	0.0514	32.8479	2.0175	3197.035
$\hat{\bar{Y}}_{MP1}$	0.1667	101.4167	6.6454	8801.253
$\hat{\bar{Y}}_{MP2}$	0.1549	97.3936	4.3906	6605.174
$\hat{\bar{Y}}_{MP3}$	0.2957	153.2938	5.5666	7684.333
$\hat{\bar{Y}}_{MP4}$	0.1607	99.3165	7.0919	9298.914
$\hat{\bar{Y}}_{MP5}$	0.2771	144.8897	5.2877	7415.407

Table 6: Percentage Relative Efficiency of Proposed Estimators.

Proposed Estimators	Population 1			Population 2		
	$\bar{y}_{srs}$	$\hat{\bar{Y}}_{R}$	$\hat{\bar{Y}}_{P}$	$\bar{y}_{srs}$	$\hat{\bar{Y}}_{R}$	$\hat{\bar{Y}}_{P}$
$\hat{\bar{Y}}_{P1}$	173.485	109.229	508.93	132.605	169.676	438.147
$\hat{\bar{Y}}_{P2}$	173.452	109.208	508.832	125.918	161.118	416.05
$\hat{\bar{Y}}_{P3}$	174.157	109.652	510.899	128.919	164.959	425.967
$\hat{\bar{Y}}_{P4}$	173.468	109.218	508.878	134.452	172.039	444.25
$\hat{\bar{Y}}_{P5}$	174.016	109.564	510.487	128.121	163.938	423.331

Efficiency comparison

In this section, compare the efficiencies proposed estimators with that of the existing estimators such as simple random sampling without replacement sample mean, ratio estimator, product estimator, modified ratio estimator and modified product estimator.

Comparison with Simple random sampling without replacement sample mean

By comparing the proposed estimators with that of simple random sampling without replacement sample mean, we arrive $\hat{\bar{Y}}_{Pi}$ is more efficient than $\bar{y}_{srs}$ only if

Table 7: Percentage Relative Efficiency of Proposed Estimators.

Proposed Estimators	Modified Ratio		Modified Product	
	Population 1	Population 2	Population 1	Population 2
$\hat{\bar{Y}}_{P1}$	105.883	120.565	316.101	303.200
$\hat{\bar{Y}}_{P2}$	108.176	101.488	303.504	216.070
$\hat{\bar{Y}}_{P3}$	105.443	108.632	479.644	257.360
$\hat{\bar{Y}}_{P4}$	107.025	127.187	309.524	324.806
$\hat{\bar{Y}}_{P5}$	102.696	106.412	452.982	246.820

Table 8: Population Parameters Simulated data.

N = 100	$\bar{Y}$ = 46.4232	$\bar{X}$ = 29.6611	C_x = 0.9440	C_y =1.0057	ρ = 0.5249
Q_1 = 6.1980	Q_3 = 43.0186	Q_r =32.8206	Q_d = 18.4180	Q_1 = 24.6084	

Table 9: Population Parameters Simulated data.

Parameters /constants	When sample size n = 5					
θ_i	0.714	0.386	0.630	0.500	0.460	0.949
λ_i	0.967	1.002	0.980	0.993	0.997	0.929
γ_i	0.957	0.976	0.962	0.970	0.972	0.944
α_i	0.787	1.015	0.819	0.897	0.933	0.731
Parameters /constants	**When sample size n=10**					
θ_i	0.714	0.386	0.630	0.500	0.461	0.949
λ_i	0.984	1.001	0.991	0.997	0.999	0.965
γ_i	0.979	0.989	0.982	0.985	0.987	0.973
α_i	0.791	1.029	0.826	0.908	0.945	0.725
Parameters /constants	**When sample size n=50**					
θ_i	0.714	0.386	0.630	0.500	0.460	0.949
λ_i	0.998	1.000	0.999	1.000	1.000	0.996
γ_i	0.998	0.999	0.998	0.998	0.998	0.997
α_i	0.794	1.040	0.831	0.917	0.954	0.721

$$V(\bar{y}_{srs}) \geq \mathrm{MSE}(\hat{\bar{Y}}_{Pi})$$

$$\rho \geq \frac{(P_i-1)^2+\delta(C_y^2(P_i^2-1)+\theta_i^2 C_x^2(P_i^2+(P_i+Q_i)(Q_i-1))}{2\delta\theta_i C_x C_y Q_i(2P_i-1)}$$

Comparison with Ratio estimator

By comparing the proposed estimators with that of ratio estimator, we arrive $\hat{\bar{Y}}_{Pi}$ is more efficient than $\bar{y}_R$ only if

$$\mathrm{MSE}(\hat{\bar{Y}}_R) \geq \mathrm{MSE}(\hat{\bar{Y}}_{Pi})$$

$$\rho \geq \frac{(P_i-1)^2+\delta(C_y^2(P_i^2-1)+C_x^2(\theta_i^2(P_i^2+(P_i+Q_i)(Q_i-1))-1)}{2\delta C_x C_y(\theta_i Q_i(2P_i-1)-1)}$$

Table 10: Biases and MSE of simulated Data.

Estimators		n=5		n=10		n=50	
		bias	MSE	Bias	MSE	bias	MSE
Existing estimators	$\bar{y}_{srs}$	-	324.163	-	153.418	-	17.059
	$\hat{\bar{Y}}_R$	4.437	391.319	2.103	185.339	0.234	20.612
	$\hat{\bar{Y}}_p$	3.116	1012.303	1.475	479.406	0.164	53.305
	$\hat{\bar{Y}}_{MR1}$	1.711	298.059	0.811	141.125	0.090	15.699
	$\hat{\bar{Y}}_{MR2}$	-0.082	260.551	-0.038	123.315	-0.004	13.713
	$\hat{\bar{Y}}_{MR3}$	1.005	277.760	0.477	131.489	0.053	14.621
	$\hat{\bar{Y}}_{MR4}$	0.332	263.390	0.158	124.671	0.018	13.864
	$\hat{\bar{Y}}_{MR5}$	0.152	261.220	0.072	123.638	0.008	13.750
	$\hat{\bar{Y}}_{MP1}$	2.243	744.331	1.062	352.479	0.118	39.208
	$\hat{\bar{Y}}_{MP2}$	1.199	499.635	0.567	236.523	0.063	26.295
	$\hat{\bar{Y}}_{MP3}$	1.949	666.781	0.923	315.688	0.103	35.090
	$\hat{\bar{Y}}_{MP4}$	1.559	574.039	0.738	271.777	0.082	30.217
	$\hat{\bar{Y}}_{MP5}$	1.421	545.009	0.673	258.001	0.075	28.677
Proposed Estimators	$\hat{\bar{Y}}_{P1}$	-4.26E-18	236.687	1.01E-17	116.973	-7.29E-18	13.478
	$\hat{\bar{Y}}_{P2}$	-1.08E-17	258.936	1.80E-17	122.142	-2.37E-17	13.544
	$\hat{\bar{Y}}_{P3}$	-1.55E-17	244.903	-1.28E-18	118.934	1.77E-17	13.504
	$\hat{\bar{Y}}_{P4}$	-1.75E-17	253.027	1.81E-17	120.805	6.42E-18	13.527
	$\hat{\bar{Y}}_{P5}$	1.07E-17	255.494	2.10E-18	121.366	9.41E-18	13.534

Comparison with Product estimator

By comparing the proposed estimators with that of product estimator, we arrive $\hat{\bar{\bar{Y}}}_{Pi}$ is more efficient than $\hat{\bar{Y}}_P$ only if

$$MSE(\hat{\bar{\bar{Y}}}_{MRi}) \geq MSE(\hat{\bar{\bar{Y}}}_{Pi})$$

$$\rho \geq \frac{(P_i-1)^2+\delta(C_y^2(P_i^2-1)+\theta_i^2C_x^2((P_i^2+(P_i+Q_i)(Q_i-1))-1)}{2\delta\theta_iC_xC_y(Q_i(2P_i-1)-1)}$$

Comparison with Modified Ratio estimator

By comparing the proposed estimators with that of ratio estimator, we arrive $\hat{\bar{\bar{Y}}}_{Pi}$ is more efficient than $\hat{\bar{Y}}_{MRi}$ only if

$$MSE(\hat{\bar{\bar{Y}}}_P) \geq MSE(\hat{\bar{\bar{Y}}}_{Pi})$$

$$\rho \geq \frac{(P_i-1)^2+\delta(C_y^2(P_i^2-1)+C_x^2(\theta_i^2(P_i^2+(P_i+Q_i)(Q_i-1))-1)}{2\delta C_xC_y(\theta_iQ_i(2P_i-1)+1)}$$

Comparison with Modified Product estimator

By comparing the proposed estimators with that of product estimator, we arrive $\hat{\bar{\bar{Y}}}_{Pi}$ is more efficient than $\hat{\bar{Y}}_{MPi}$ only if

$$MSE(\bar{y}_{MPi}) \geq MSE(\hat{\bar{\bar{Y}}}_{Pi})$$

$$\rho \geq \frac{(P_i-1)^2+\delta(C_y^2(P_i^2-1)+C_x^2(\theta_i^2(P_i^2+(P_i+Q_i)(Q_i-1))-1)}{2\delta\theta_iC_xC_y(Q_i(2P_i-1)+1)}$$

Where $P_i=\alpha_i\lambda_i+(1-\alpha_i)\gamma_i$, and $Q_i=\alpha_i\lambda_i-(1-\alpha_i)\gamma_i$

Numerical Study

In this section we consider two natural populations, for accessing the performance of the proposed estimators. The computed values of constants and parameters of these populations are given below:

Population 1: Cochran [2] page 325

N = 10 n=3 $\bar{Y}$ = 101.1 $\bar{X}$ = 58.8

ρ =0.6515 S_x =7.9414 S_y =15.4448

C_x = 0.1350 C_y =0.1527 β_1 =0.2363

β_2 =2.2388 Q_1 = 53 Q_3 = 61.5

Q_r =8.5 Q_d =4.25 Q_a = 57.25

Population 2: Singh and Chaudhary [18], page 177

N = 34 n=5 $\bar{Y}$ =199.4412 $\bar{X}$ = 856.4117

ρ = 0.4453 S_x =733.1407 S_y =150.2150

C_x = 0.8561 C_y =0.7532 β_1 =7.955 β_2 =13.3667

Q_1 = 402.5 Q_3 = 1049 Q_r = 646.5

Q_d =323.25 Q_a = 725.75

These values are obtained when $a_i=B(\hat{\bar{\bar{Y}}}_{MRi}), b_i=B(\hat{\bar{\bar{Y}}}_{MPi})$ and α_i (i = 1,2,3,4,5) is optimum and these parameters are used to obtained the biases and mean squared errors of the proposed and existing estimators, and also compare the percentage relative efficiency of proposed estimators with that of the existing estimators such as simple random sampling sample mean, ratio, product estimators, modified ratio and modified product estimators with quartiles and their functions (Tables 8, 9 and 10).

Simulation Study

In this section a finite population of size N= (100,100) is generated from a bivariate normal distribution with means $\mu_{1=32}$, $\mu_{1=50}$ standard deviations σ_1 = 28.3, σ_1 = 41.3 and correlation coefficient ρ =0.24. The simulation process is repeated under 10000 times. The average value of the biases ,mean squared errors and percentage relative efficiencies with respect to the existing and proposed estimators are obtained for a random sample of size n = 5, n = 10, and n= 50 are drawn by SRSWOR. The parameter values, biases, mean squared errors and the percentage relative efficiencies are given in the following tables 11 and 12.

Table 11: Percentage relative efficiency of simulated data.

Proposed estimators	Sample size n=5			Sample size n=10			Sample size n=50		
	$\bar{y}_{srs}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_P$	$\bar{y}_{srs}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_P$	$\bar{y}_{srs}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_P$
$\hat{\bar{Y}}_{P1}$	136.96	165.33	427.70	131.16	158.45	409.84	126.57	152.93	395.50
$\hat{\bar{Y}}_{P2}$	125.19	151.13	390.95	125.61	151.74	392.50	125.96	152.19	393.57
$\hat{\bar{Y}}_{P3}$	132.36	159.78	413.35	129.00	155.83	403.09	126.33	152.64	394.75
$\hat{\bar{Y}}_{P4}$	128.11	154.66	400.08	127.00	153.42	396.84	126.11	152.38	394.06
$\hat{\bar{Y}}_{P5}$	126.88	153.16	396.21	126.41	152.71	395.01	126.04	152.30	393.85

Table 12: Percentage relative efficiency of simulated data.

Proposed Estimators	Modified Ratio Estimators			Modified Product Estimators		
	n=5	n=10	n=50	n=5	n=10	n=50
$\hat{\bar{Y}}_{P1}$	125.93	120.65	116.48	314.48	301.33	290.91
$\hat{\bar{Y}}_{P2}$	100.62	100.96	101.25	192.96	193.65	194.15
$\hat{\bar{Y}}_{P3}$	113.42	110.56	108.28	272.26	265.43	259.86
$\hat{\bar{Y}}_{P4}$	104.10	103.20	102.49	226.87	224.97	223.38
$\hat{\bar{Y}}_{P5}$	102.24	101.87	101.59	213.32	212.58	211.89

Conclusion

In this paper, we have proposed a class of modified ratio cum product estimators for the estimation of finite population mean of the study variable Y with known quartiles and their functions of the auxiliary variable. The bias, mean squared error and the conditions under which the proposed estimator perform better than the existing estimators like SRSWOR sample mean, ratio estimator, the corresponding modified ratio estimators and modified product estimators have been derived. The performances of the proposed estimators for some known natural populations are also observed. The proposed estimator has less bias (almost unbiased) and means squared error than all the existing estimators. Figure 1 and 2 are showing the mean squared errors of proposed and existing estimators. In this case PREs are in general ranging from 101.488 to 510.899A simulated data from Bivariate normal population have been used to assess the performances of the proposed estimators. It is observed that the PREs are in general ranging from 100.62 to 427.70.It shows that proposed class of estimators performing better than all the existing estimators.

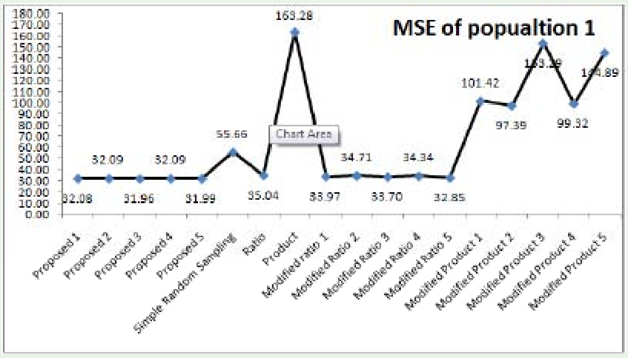

Figure 1: MSE of Proposed and Existing Estimators of Population 1.

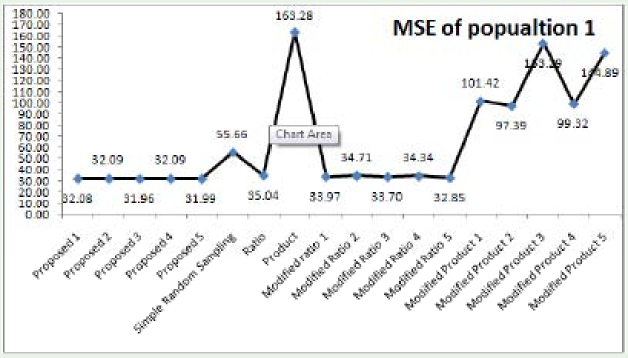

Figure 2: MSE of Proposed and Existing Estimators of Population 2.

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