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Short Communication

A Generalization of the Exponential Transmuted Exponential Distribution Arising from Box-Cox Transformation

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Abstract

The Exponential Transmuted Exponential distribution (ETE) appeared in [1] and in this paper we present a new generalization of the ETE distribution based on a Box-Cox transformation of the form

$$Z = \frac{X^{\lambda} - (\mu\lambda + 1) - \sigma\lambda\mu}{\sigma^2\lambda\mu}$$

Where $0 \neq \lambda, \mu, \sigma \in R, \sigma^2 > 0,$ and Z is ETE distributed. We also show the new distribution is a good fit to some real-life data, indicating practical significance.

Result

Lemma 2.1. If Z is ETE distributed, then the random variable

$$X = \left(Z\sigma^2\lambda\mu + \lambda\mu + 1 + \sigma\lambda\mu\right)^{\frac{1}{\lambda}}$$

Has CDF $\quad F_{\theta,\beta,\gamma}\dfrac{\left(\chi^{\lambda} - (\mu\lambda + 1) - \sigma\lambda\mu\right)}{\sigma^2\lambda\mu}$

Where $F_{\theta,\beta,\gamma}(.)$ is the CDF of the ETE distribution, that is, $F_{\theta,\beta,\gamma}(.) = 1 - e^{-\theta,\beta(.)}\left(1 - \gamma + \gamma e^{-\beta(.)}\right)^{\theta}$

Proof. Since Z is ETE distributed, then,

$$P(Z \le z) = 1 - e^{-\theta\beta z\left(1-\gamma+\gamma e^{-\beta z}\right)^{\theta}}$$

$$= F_{\theta,\beta,\gamma}(z)$$

Thus by the transformation technique

$$P(X \le x) = p\left(Z \le \frac{X^{\lambda} - (\mu\lambda + 1) - \sigma\lambda\mu}{\sigma^2\lambda\mu}\right)$$

$$= 1 - e^{-\theta\beta\left(\frac{X^{\lambda}-(\mu\lambda+1)-\sigma\lambda\mu}{\sigma^2\lambda\mu}\right)}\left(1 - \gamma + \gamma e^{-\beta\left(\frac{X^{\lambda}-(\mu\lambda+1)-\sigma\lambda\mu}{\sigma^2\lambda\mu}\right)}\right)^{\theta}$$

$$= F_{\theta,\beta,\gamma}\left(\frac{\chi^{\lambda} - (\mu\lambda + 1) - \sigma\lambda\mu}{\sigma^2\lambda\mu}\right)$$

Application

Remark 3.1: If a random variable X has CDF given by the previous Lemma, write..

It was observed that the following combination of parameter values was very good in fitting the data on patients with breast cancer, Section 5.2 [1].

$$\left(\hat{\theta}, \hat{\beta}, \hat{\gamma}, \hat{\lambda}, \hat{\mu}, \hat{\sigma}\right) = (1.94227, 0.0738399, -1.11855, 0.725426, 0.300217, 2.14699)$$

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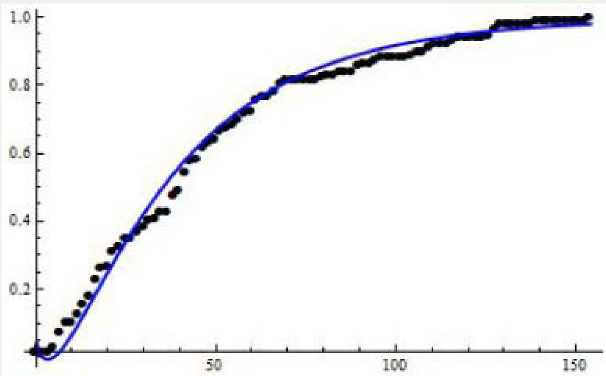

Figure 1: The CDF of *BCETE* (1.94227, 0.0738399, -1.11855, 0.725426, 0.300217, 2.14699) fitted to the empirical distribution of the data on patients with breast cancer, Section 5.2 [1].

Further Developments

As a further development, we propose a so-called Polynomial Transmuted-BCETE distribution inspired by [2]. Let Np be binomial with parameters $n \in \mathrm{N} = \{1,2,3,\ldots\}$ and $p \in [0,1]$ shifted up by one, then the probability generating function (PGF) for $s \in [0,1]$ is

$$G_p(s) = s(1-p+ps)^n$$

if $\alpha \in [-1,0]$, then $G_{-\alpha}(F(x)) = F(x)(1+\alpha-\alpha F(x))^n$

and if $\alpha \in [0,1]$, the survival function is $s_\alpha(x) = S(x)(1-\alpha+\alpha S(x))^n$

Where $S(x) = 1-F(x)$. thus we have the following

Theorem

If a random variable X has CDF F(x), and then the CDF of the Polynomial-Transmuted-X distribution is given by

$$L(x;\alpha,n) = \begin{cases} F(x)(1+\alpha-\alpha F(x))^n \text{ for } \alpha \in [-1,0] \text{ and } n \in \mathrm{N} \\ 1-\left[(1-F(x))(1-\alpha+\alpha(1-F(x)))^n\right] \text{ for } \alpha \in [1,0] \text{ and } n \in \mathrm{N} \end{cases}$$

If X has CDF given by Lemma 1.1, then we have the following

Corollary

The CDF of the Polynomial Transmuted-BCETE distribution is given for $\alpha \in [-1,0]; n \in \mathrm{N}$

By $w(x;\alpha,n,\theta,\beta,\lambda,\mu,\sigma) = F_{\theta,\beta,\gamma}\left(\frac{\chi^\lambda-(\mu\lambda+1)-\sigma\lambda\mu}{\sigma^2\lambda\mu}\right)\left\{1+\alpha-\alpha F_{\theta,\beta,\gamma}\left(\frac{\chi^\lambda-(\mu\lambda+1)-\sigma\lambda\mu}{\sigma^2\lambda\mu}\right)\right\}^n$

and for $\alpha \in [0,1]; n \in \mathrm{N}$ by

$$w(x;\alpha,n,\theta,\beta,\lambda,\mu,\sigma) = 1-\left[\left\{1-F_{\theta,\beta,\gamma}\left(\frac{\chi^\lambda-(\mu\lambda+1)-\sigma\lambda\mu}{\sigma^2\lambda\mu}\right)\right\}\left\{1-\alpha+\alpha\left(1-F_{\theta,\beta,\gamma}\left(\frac{\chi^\lambda-(\mu\lambda+1)-\sigma\lambda\mu}{\sigma^2\lambda\mu}\right)\right)\right\}^n\right]$$

Where $F_{\theta,\beta,\gamma}(.)$ is the CDF of the ETE distribution, that is,

$$F_{\theta,\beta,\gamma}(.) = 1-e^{-\theta,\beta(.)}\left(1-\gamma+\gamma e^{-\beta(.)}\right)^\theta$$

Concluding Remarks

Our hope is that the new class of distributions presented in this paper will find application in cancer modeling and forecasting.

References

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2. Kozubowski TJ, Podgorski K. Transmuted distributions and random extrema. Statistics and Probability Letters. 2016; 116: 6-8.